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UNSTEADY HEAT AND MASS TRANSFER FROM SPHERES

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Abstract—This paper reports theoretical and experimental results on unsteady heat and mass transfer from small spherical bodies for small Strouhal numbers. Experiments are done in a stream consisting of steady and unsteady components. Unsteady flows are generated by an acoustic resonance in a tube and fluctuate purely sinusoidally. In heat-transfer experiments in an air stream several spherical thermistors are used and the time-averaged Nusselt number is shown to be well correlated, both experimentally and theoretically, by means of Reynolds number with time-averaged velocity as its characteristic velocity and the ratio of velocity amplitude of unsteady component to steady component.

Unsteady mass-transfer experiments are done by evaporation of small droplets of water and prophyl-alcohol in an air stream. The results are found to be represented by a similar correlating relation to that for heat transfer. A resonant oscillation of deformation of a droplet is observed due to surface tension and during this resonance the Sherwood number is found to increase proportionally to the unsteady component of the stream.

NOMENCLATURE

C, concentration [kg/m^3];
D, diameter of a sphere [m];
D_v, diffusivity of vapor in the air [m^2/h];
F, surface area [m^2];
h, heat-transfer coefficient [kcal/m^{2}h°C];
k, mass-transfer coefficient [m/h];
Nu, Nusselt number, hD/λ;
Pr, Prandtl number, ν/α;
P, pressure [kg/m^2];
Q, heat [kcal];
R, gas constant [kgm/kg°K];
Re, Reynolds number, UD/ν;
S, Strouhal number, $\omega D/U$;
Sc, Schmidt number, ν/D_v;
Sh, Sherwood number, kD/D_v;
T, temperature [°K];
t, time [h];
U_1, amplitude of the fluctuant velocity [m/h];
U_0, uniform velocity [m/h];
u, velocity [m/h].

Greek symbols

α, thermal diffusivity [m^2/h];
γ, specific weight [kg/m^3];
ε, dimensionless amplitude of the fluctuant velocity, U_1/U_0;
λ, thermal conductivity [kcal/mh°C];
ν, kinematic viscosity [m^2/h];
ρ, density [kgh^2/m^4];
σ, surface tension [kg/m];
ω, angular velocity [1/h].

Subscripts

a, in the main flow;
l, lead wire;
m, mean value between the surface and the main flow;
0, steady term;
s, values under a steady state condition;
v, values under an unsteady state condition;
$\bar{}$, time-averaged values;
$'$, fluctuant values.

1. INTRODUCTION

STUDIES of heat and mass transfer from a sphere and a circular cylinder have long been considered one of the fundamental problems in heat and mass transfer. But the reports on heat and mass transfer from small particles are comparatively few. Experimental studies on this problem were made formerly by Flössling and Yuge, recently by Tsubouchi [1].

All these studies were done under a steady state condition. Under an unsteady condition, it has been said that, in general, the heat- and mass-transfer rate increases, but systematic studies have never been done. Therefore, the purpose of this paper is to study the effect of an unsteady flow, which is composed of a uniform flow and a periodically fluctuating flow approximated by a sinusoidal wave, on heat and mass transfer from small spheres. As the first step of this study, we experimentally and theoretically investigate the time-averaged heat- and mass-transfer rate from spheres in such a flow.

Previous works on this problem were done by Zijnen [2], who vibrated a fine wire of 5 μ in the direction of the flow, by Ramachandran [3], [4] and by Bayley [5]. Their works were made for a fine wire and the experimental methods and treatments were various, but only particular cases were investigated.

In this report, we use spherical particles and spherical liquid drops and investigate the effect of the fluctuant velocity field on heat and mass transfer and the associated phenomena. In the experiment, we use thermistors in the heat-transfer experiment and water and prophyl-alcohol in the mass-transfer experiment.

2. EXPERIMENTAL APPARATUS AND METHOD

2.1 *Experimental apparatus*

In this study to generate a large fluctuant velocity, we use an acoustic standing wave in a pipe as shown in Fig. 1. The apparatus is composed of a pipe system and a blower, which sends a uniform flow. The pipe and a reciprocating piston fitted on one end of the pipe, consist of the pipe system. The other end of the pipe is open and connected to the blower through the surge tank.

When we vibrate the piston at the frequency of $f = 3c/4L$, (where c is the sound velocity and L is the length of the pipe), we can generate a standing sound wave which has the pressure nodes at the open end and at the point of $2L/3$ distance from ent entrance. If we open this point to the free air, there is no effect on the sound wave. Therefore, we make a hole at the $2L/3$ point and send in the steady uniform flow from the blower at the open end of the pipe and out at the $2L/3$ point. By superposing the

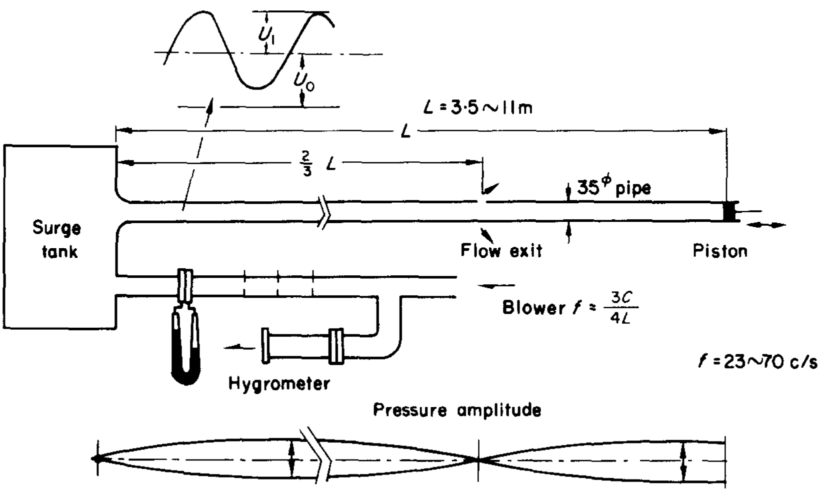

FIG. 1. Experimental apparatus.

uniform flow to the standing sound wave, we can get the flow as shown in Fig. 2 in the vicinity of the open end of the pipe. It can be seen that the fluctuant velocity component is exactly approximated by the sinusoidal wave. The piston is vibrated by a crank mechanism and the frequency is changed by a speed change gear. The resonant frequency can be changed by varying the total length of the pipe and the amplitude of the fluctuant velocity can be changed by the piston stroke. In this experiment, we vary the total length of the pipe from 3·5 to 11 m and the piston stroke from 1 to 12 mm. The frequency range in the experiment is 23–70 c/s and that of the fluctuant velocity amplitude is below 6 m/s. The maximum point of the velocity amplitude is at the open end and exit of the flow. We set the measurement section at the open end, because its velocity distribution is almost uniform in the cross section except in the neighborhood of the pipe wall. The time-averaged velocity is measured by an orifice-meter which is between the surge tank and the blower, and the fluctuation of the velocity is measured by a hot wire anemometer of constant temperature. A part of the flow from the blower is divided and lead through the valve to the hygrometer to measure the water vapor pressure of the flow.

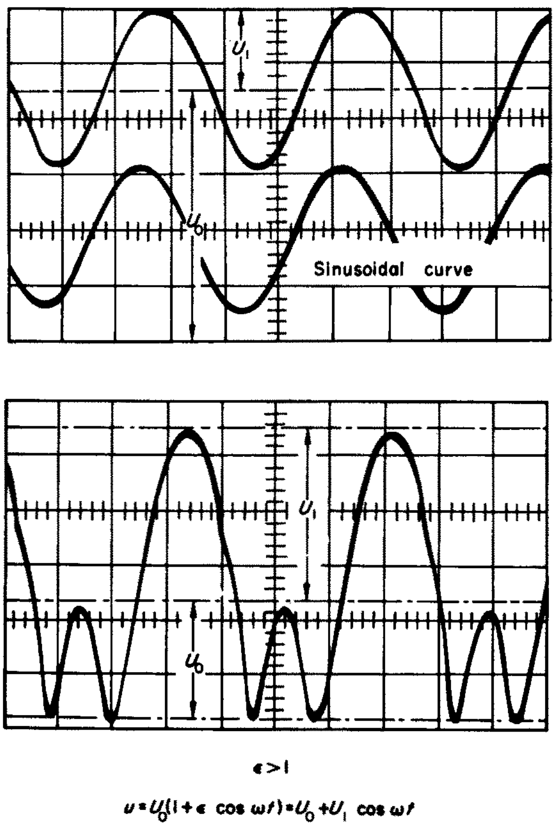

FIG. 2. Velocity fluctuation.

2.2 *The experiment on heat transfer from spherical particles*

The sketch of the measuring part for heat transfer is shown in Fig. 3. We use bead type thermistors as tested spheres and it is held at the center of the pipe by the lead wires from the pipe wall. We measure the electric power input to a thermistor and calculate the total heat input and the surface temperature. The used thermistors are neither perfect spheres nor similar figures to each other, therefore as a reference length, we adopt the diameter of a sphere with the same surface area as the sample. The thermistors used in this experiment are shown in Fig. 4.

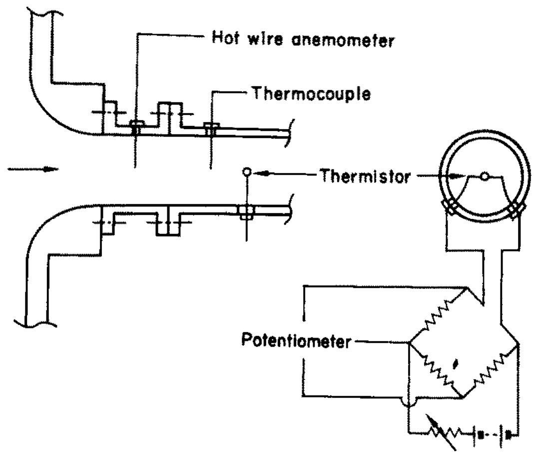

FIG. 3. Test section for heat transfer.

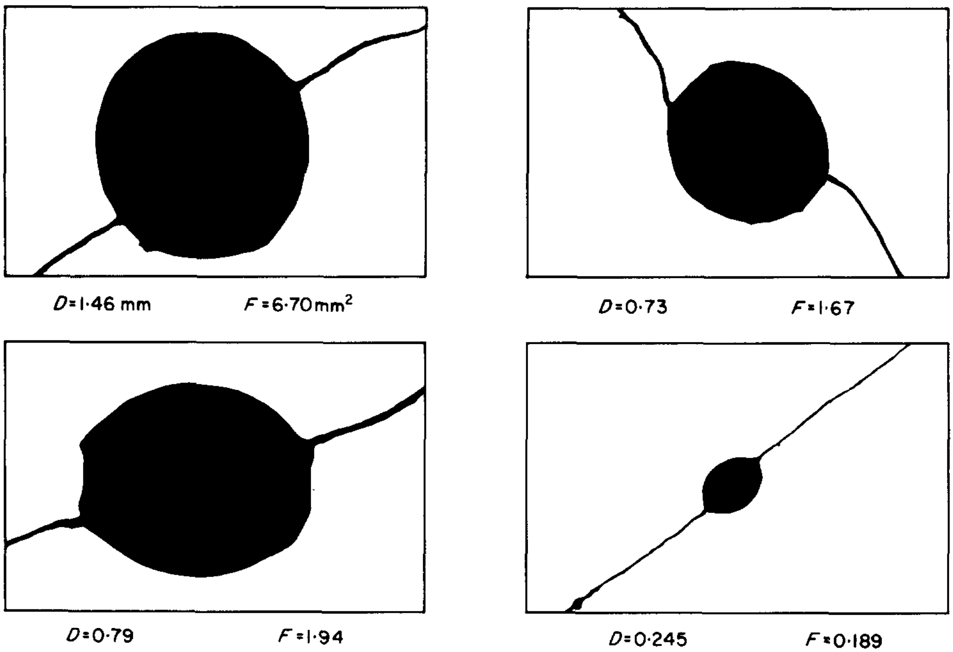

FIG. 4. Thermistors used in the experiment.

2.2.1 *The heat-transfer coefficient.* The heat-transfer coefficient h is defined as follows:

$$Q_C = hF(T_w - T_a) \tag{1}$$

where Q_C is heat transferred by convection, F is the surface area of the sample, and T_w, T_a are the temperatures of the surface and the main stream, respectively. The heat transferred by convection is obtained by subtracting the heat by conduction from the lead wires and the radiant heat of the surface from the electrical heat input. The surface temperature is calculated from the measured electrical resistance of the sample.

When we define the time-averaged term and the fluctuant term of the heat-transfer coefficient as $\bar{h}$ and h' and those of the surface temperature as $\bar{T}_w$ and T', the heat transferred by convection Q_C is expressed in the following form:

$$Q_C = (\bar{h} + h')(\bar{T}_w + T' - T_a)\,F. \tag{2}$$

To measure the mean value of $\bar{Q}_C$, we take the time-average of Eq. (2). Then:

$$\bar{Q}_C = \{\bar{h}(\bar{T}_w - T_a) + \overline{h'T'}\}\,F. \tag{3}$$

In this study, the fluctuation of the surface temperature almost is negligible as the time-constant of the sample is very large, so $\bar{h}$ is obtained from the following equation:

$$\bar{Q}_C = \bar{h}(\bar{T}_w - T_a)\,F. \tag{4}$$

2.2.2 *The heat transferred by convection.* The heat transferred by convection Q_C is obtained from the following equation:

$$Q_C = Q_T - (Q_l + Q_R) \tag{5}$$

where Q_l is the heat transferred by conduction from the lead wires, Q_R the heat transferred by radiation from the surface of the sample and Q_T the total heat input.

The total Joule heating Q_T is described as:

$$Q_T = 0{\cdot}86\ EI \tag{6}$$

where I is the electric current and E is the voltage between both ends of lead wires. Q_R is calculated as follows:

$$Q_R = C_b \cdot e(\overline{T}_w^4 - T_a^4) \tag{7}$$

where C_b is $4{\cdot}88 \times 10^{-8}$ kcal/m²h°K and the emissivity e of the surface is taken as 0·5. Q_l is calculated by the following heat balance equation:

$$\frac{1}{\alpha_l}\frac{\partial T_l}{\partial t} + \frac{\partial^2 T_l}{\partial x^2} = \frac{4h_l}{\lambda_l d}(T_l - T_a) - \frac{0{\cdot}86\, I^2 r_0}{\lambda(\pi d^2/4)}(1 + a\,T_l) \tag{8}$$

where T_l is the temperature at an arbitrary point of the lead wire, α_l the thermal diffusivity, λ_l the thermal conductivity, r_0 the electrical resistance per unit length at 0°C, a the coefficient of resistance, d the diameter of the lead wire and h_l is the heat-transfer coefficient of the lead wire. The velocity of the flow is described as follows:

$$u = U_0 + U_1 \cos \omega t = U_0(1 + \varepsilon \cos \omega t). \tag{9}$$

The lead wire is fine, so the heat-transfer coefficient h_l may be expressed as follows:

$$h_l = 0{\cdot}841\,\frac{\lambda}{d}\,[Re_{ol}(|1 + \varepsilon \cos \omega t|)]^{0{\cdot}36} = \bar{h}_l + h_{l1}\cos \omega t \tag{10}$$

where Re_{ol} is Reynolds number and the sign of the absolute value means that h_l is independent of the direction of the velocity. Neglecting the non-linear term and making an approximate calculation, the following equation is obtained:

$$\overline{T}_l - T_a = (\overline{T}_w - T_a)\,e^{-\mu_0 x}\left[1 + \frac{\mu_1^4}{16\mu_0^3}x + \frac{\mu_1^4}{16\mu_0^2}x^2\right]\times e^{-\mu_0 x} \tag{11}$$

where $\mu_0^2 = 4\bar{h}_l/\lambda_l d$, $\mu_1^2 = 4h_{l1}/\lambda_l d$.

Ignoring the small terms of equation (11), the heat loss from the lead wire Q_{lv} is expressed as follows:

$$Q_{lv} = \frac{\pi d^2}{2}(\overline{T}_w - T_a)\,\mu_0 \lambda_l. \tag{12}$$

On the other hand, the heat transferred by conduction from the lead wire without the fluctuant velocity Q_{ls} is expressed as:

$$Q_{ls} = \frac{\pi d^2}{2}(\overline{T}_w - T_a)\,\mu_s \lambda_l \tag{13}$$

where $\mu_s^2 = 4h_s/\lambda_l d$ and h_s is the heat-transfer coefficient for a steady flow. Finally, Q_{lv} is represented as follows:

$$Q_{lv} = Q_{ls}[\overline{(|1 + \varepsilon \cos \omega t|)^{0{\cdot}36}}]^{\frac{1}{2}}. \tag{14}$$

2.2.3 *The surface temperature of the sample.* There is a correlation between the electrical resistance and the temperature of a thermistor and this relation is obtained from the measurement in a constant temperature box for a uniform internal temperature of a sample. But in an experiment of unsteady heat transfer, the temperature of the sample is not uniform because of a heat flux to the surface, and the temperature obtained by the use of the measured correlation is expected to the temperature of the core region of the sample. Therefore, we have to calculate the internal temperature distribution and the surface temperature by assuming the Joule heating distribution inside the sample. Assuming the spherical symmetry of the heating source, we may define the temperature distribution as the function of the radius and the time. If the distribution of the heat production $q(r)$ is independent of the time, the heat balance equation inside the sample is obtained as follows:

$$\frac{1}{\alpha_t}\frac{\partial T}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{\lambda_t}q(r) \tag{15}$$

where α_t and λ_t are the thermal diffusivity and the thermal conductivity of the sample, respectively. So as to take into account the effect of the unsteady state, we assume that the heat-transfer coefficient h and the surface temperature T_w are given as follows:

$$\left.\begin{aligned} h &= \bar{h} + h_1 \cos \omega t \\ T_w - T_a &= T_0 + T_1 \cos(\omega t + \delta) \end{aligned}\right\} \tag{16}$$

where δ is the phase difference. Assuming a quasi-steady state as stated later, the heat transferred from the surface is denoted in the following form:

$$Q_c = \{\bar{h}T_0 + \tfrac{1}{2}h_1 T_1 \cos\delta) + (h_1 T_0 + \bar{h}T_1 \cos\delta)\cos\omega t - \bar{h}T_1 \sin\delta \sin\omega t\} F \tag{17}$$

where F is the surface area of the sample. Dividing T into the steady component Θ_0, and the unsteady component Θ_1; then, for Θ_0 and Θ_1, the following equations are obtained:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\Theta_0}{dr}\right) + \frac{1}{\lambda_t} q(r) = 0 \tag{18}$$

$$\frac{1}{\alpha_t}\frac{\partial \Theta_1}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \Theta_1}{\partial r}\right). \tag{19}$$

The boundary conditions at $r = D/2$ are expressed as:

$$\left.\begin{aligned} &\Theta_0 = T_0 + T_a, \\ &-\lambda_t \left(\frac{\partial \Theta_0}{\partial r}\right) = \bar{h}T_0 + \tfrac{1}{2} h_1 T_1 \cos\delta \\ &\Theta_1 = T_1 \cos\omega t, \\ &-\lambda_t \left(\frac{\partial \Theta_1}{\partial r}\right) \\ &\quad = (h_1 T_0 + \bar{h}T_1 \cos\delta)\cos\omega t - \bar{h}T_1 \sin\delta \sin\omega t. \end{aligned}\right\} \tag{20}$$

Taking into consideration the heating characteristics of the thermistor, $q(r)$ is assumed as follows:

$$q(r) = \frac{A_0}{r}\sin\frac{2\pi}{D} r \tag{21}$$

where A_0 is an arbitrary constant.

$$\Theta_0 = T_0 + T_a + \frac{A_0 D^2 1}{4\pi^2 \lambda_t\, r}\sin\frac{2\pi}{D} r \tag{22}$$

where

$$A_0 = \frac{1}{D^2}\{\bar{h}T_0 + \tfrac{1}{2}h_1 T_1 \cos\delta\} F. \tag{23}$$

From equation (19),

$$\Theta_1 = \frac{A}{r}[e^{nr}\cos(\omega t + nr + \Delta) - e^{-nr}\cos(\omega t - nr + \Delta)],$$

$$n = \sqrt{\frac{\omega}{2\alpha_t}} \tag{24}$$

The constants A, Δ and δ, are obtained by following simultaneous equations, which are obtained from the boundary conditions.

$$h_1 T_0 + \bar{h}T_1 \cos\delta = X, \qquad -\bar{h}T_1 \sin\delta = Y \tag{25}$$

At $2R = D$,

$$\left.\begin{aligned} T_1 \cos\delta &= \frac{A}{R}\{e^{nR}\cos(nR + \Delta) - e^{-nR}\cos(nR - \Delta)\} \\ T_1 \sin\delta &= \frac{A}{R}\{e^{nR}\sin(nR + \Delta) + e^{-nR}\sin(nR - \Delta)\} \end{aligned}\right\} \tag{26}$$

$$\left.\begin{aligned} X =& \left[\frac{\lambda_t A}{R^2}(e^{nR} - e^{-nR})\cos nR - \frac{n\lambda_t A}{R}\{(e^{nR} + e^{-nR})\cos nR - (e^{nR} - e^{-nR})\sin nR\}\right]\cos\Delta \\ &+ \left[-\frac{\lambda_t A}{R^2}(e^{nR} + e^{-nR})\sin nR - \frac{n\lambda_t A}{R}\{-(e^{nR} - e^{-nR})\sin nR - (e^{nR} + e^{-nR})\cos nR\}\right]\sin\Delta \\ Y =& \left[-\frac{\lambda_t A}{R^2}(e^{nR} + e^{-nR})\sin nR - \frac{n\lambda_t A}{R}\{-(e^{nR} + e^{-nR})\cos nR - (e^{nR} - e^{-nR})\sin nR\}\right]\cos\Delta \\ &- \left[\frac{\lambda_t A}{R^2}(e^{nR} - e^{-nR})\cos nR - \frac{n\lambda_t A}{R}\{(e^{nR} + e^{-nR})\cos nR - (e^{nR} - e^{-nR})\sin nR\}\right]\sin\Delta. \end{aligned}\right\} \tag{27}$$

Assuming a quasi-steady state, the maximum values of T_1 and δ within the region of this experiment are

$$T_1 = 0{\cdot}17\,T_0, \qquad \delta = 105°. \tag{28}$$

Therefore, the effect of the second term of equation (3) is less than 2 per cent and neglibible. Denoting the core temperature of the sample as T_c the relation between T_c and T_0 is found from equation (22). Using the value of T_c obtained from the resistance, we can get the time-averaged surface temperature T_0 as:

$$\frac{1}{T_c - T_a}\left(1 + \frac{\bar{h}D}{2\lambda_t}\right) = \frac{1}{T_0} \tag{29}$$

For the steady state:

$$\frac{1}{T_c - T_a}\left(1 + \frac{h_s D}{2\lambda_t}\right) = \frac{1}{T_w - T_a} \tag{30}$$

where h_s is h for $u = U_0$. Using equation (42) resulted from this experiment, the relation between h_s and $\bar{h}$ is expressed as follows:

$$\bar{h} = \frac{\lambda_t}{D}\left[2 + a_0\left(\frac{D}{\lambda}h_s - 2\right)\right] \tag{31}$$

where a_0 is the value dependent on ε only.

2.3 *The experiment of mass transfer from liquid drops*

In the case of mass transfer, we need to get the mass flux of the evaporation and the concentration of the vapor molecule at the surface, corresponding to the heat flux by convection and the surface temperature. The apparatus for mass transfer is shown in Fig. 5. A liquid drop is fixed at the center of the pipe by the 30 μ Cu–Constantan thermocouple. The vapor concentration on the drop's surface is obtained by measuring the temperature of the liquid drop by a thermocouple. There are two windows, one over and the other below the liquid drop. The variation of the diameter with time is measured at regular intervals by a photomicroscope. A stroboscope is used to light the drop.

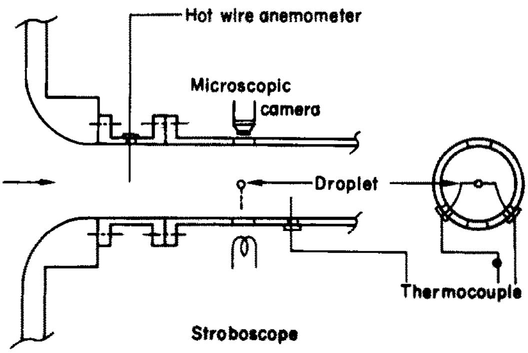

FIG. 5. Test section for mass transfer.

In this experiment, water and prophyl-alcohol are used as liquid and each physical constant is obtained from the International Critical Tables. The liquid drop is not a perfect sphere but can be sufficiently treated as a sphere. Therefore, we use the average diameter as the reference length. An example of the variation of the liquid drop diameter with time shown in Fig. 6.

2.3.1 *The mass-transfer coefficient.* The mass-transfer coefficient k is generally defined as follows:

$$I_f = kF(C_0 - C_a) \tag{32}$$

where I_f is the flux from the drop, F the surface area and C_0 and C_a are the concentrations on

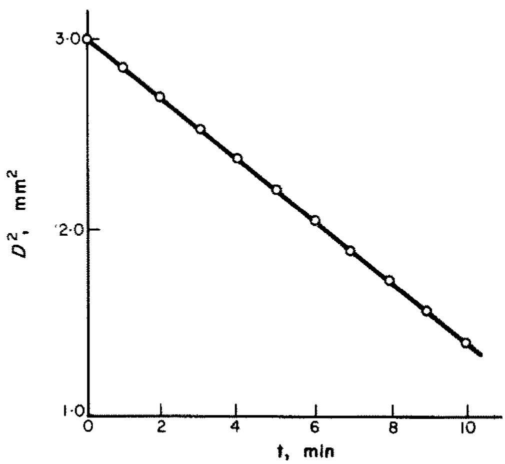

FIG. 6. Variation of liquid drop diameter with time.

the surface of the drop and in the main flow, respectively. Assuming that the vapor can be treated as an ideal gas and the temperature difference between the surface of the sample and the main flow is small, then $C_0 = P_0/RT_m$ and $C_a = P_a/RT_m$ are calculated from the pressure of the main flow and P_0 at the surface of the sample, while R is the gas constant of the vapor and T_m the mean absolute temperature between the main flow and the surface. If D is the diameter of the liquid drop and γ is the specific weight of the liquid, we have

$$I_f = -\mathrm{d}\left(\frac{\pi}{6}D^3\gamma\right)/\mathrm{d}t$$

Substituting these equations into Equation (32), the mass-transfer coefficient k is expressed as follows:

$$k = \frac{R \cdot \gamma \cdot T_m}{2(P_0 - P_a)}\left(-\frac{\mathrm{d}D}{\mathrm{d}t}\right). \qquad (33)$$

Total pressure P around the liquid drop remains constant, so the pressure distribution of the air is opposite to that of the liquid vapor. The liquid drop supported by the thermocouple is not a perfect sphere, so we must check this effect in the calculation of the mass transfer coefficient. Regarding the shape of the liquid drop as a rotating ellipse and varying the ratio of the each axial lengths a/b under a constant volume condition, the variation of the mass transfer coefficient is within 1·0 per cent for a/b between 0·8 and 1·2. When the liquid drop is not under a resonance condition, as denoted later, a/b in this experiment is almost 1·0, so the effect of it can be ignored.

In the case of mass transfer as well as heat transfer, the time-averaged value of equation (33) is important. Then the time-averaged mass-transfer coefficient is obtained as follows:

$$\bar{k} = \overline{\frac{R \cdot \gamma \cdot T_m}{2(P_0 - P_a)}}\left(-\frac{\overline{\mathrm{d}D}}{\mathrm{d}t}\right). \qquad (34)$$

2.3.2 *The partial pressure of the liquid vapor in the main flow and at the surface of the liquid drop.* It is necessary to know the partial pressure of the liquid vapor at the surface of the drop and that in the main flow. Vapor is in the saturated condition at the surface, therefore, the vapor pressure at the surface depends only on the surface temperature. The liquid drop is supported with the two lead wires of the thermocouple and changing the diameters of the wires, we get the same temperature, so it is known that the temperature distribution inside the liquid drop is almost uniform.

This phenomenon can be explained by the existance of the convective flow inside the liquid drop. Therefore, the temperature measured by the thermocouple fixing the liquid drop is used as the surface temperature of the drop.

The vapor pressure in the main flow is assumed to be zero for prophyl-alcohol, but for water we must measure it accurately, and the following method is used. Ten pairs of Cu–Constantan thermocouple of 100 μ are connected to each other in series on a mica plate with a 0·2 mm thickness and one end of the thermocouples is wet by covering with gauze and the other is dry and exposed to the air flow. When the dry and wet bulb temperature are described as θ and θ_w, respectively, the vapor pressure in the air is calculated from the Sprung's equation:

$$P_a = P_w - AP(\theta - \theta_w)/755 \qquad (35)$$

where P_w (kg/m^2) is the saturated vapor pressure at the temperature θ_w(°C) and P (kg/m^2) is the total pressure. The velocity is more than 3 m/s, so we adopt $A = 0·5$.

3. THE EXPERIMENTAL RESULTS

We transform the experimental results into nondimensional forms. For heat and mass transfer in an unsteady flow, dimensionless transfer coefficients are generally expressed as:

$$Nu = f(Re_0, Pr, S, \varepsilon) \qquad (36)$$

$$Sh = f(Re_0, Sc, S, \varepsilon) \qquad (37)$$

where the Nusselt number is $Nu = hD/\lambda$, the Reynolds number $Re_0 = U_0D/\nu$, the Strouhal number $S = \omega D/U_0$, the ratio of the fluctuant velocity component to that of the steady one $\varepsilon = U_1/U_0$, the Sherwood number $Sh = kD/D_v$, the Schmidt number $Sc = \nu/D_v$ and the Prandtl number $Pr = \nu/\alpha$. In this experiment the Prandtl number is 0·72 for the air and the Schmidt number is 0·60 for water and 1·60 for prophyl-alcohol. The physical properties are used at the average temperature $T_m = (T_w + T_a)/2$ in equations (36) and (37).

3.1 *The experimental results for steady states*

In order to make sure of the accuracy of this experimental method, we first make experiments under a steady state condition. These results are shown in Fig. 7. We take the value Nu_0 or Sh_0 as the vertical axis and the value $Pr^{\frac{1}{3}}Re_0^{\frac{1}{2}}$ or $Sc^{\frac{1}{3}}Re_0^{\frac{1}{2}}$ as the horizontal axis. These experimental results coincide very well with those of Ranz and Marshall [6] and Tsubouchi [1] made by a similar method. As it is well known that the Nusselt number and the Sherwood number are 2·0 for the small Re region, we take the following emperical formula for these results:

$$\left.\begin{aligned} Nu_0 &= 2{\cdot}0 + 0{\cdot}55\, Pr^{\frac{1}{3}} Re_0^{\frac{1}{2}} \\ Sh_0 &= 2{\cdot}0 + 0{\cdot}55\, Sc^{\frac{1}{3}}\, Re_0^{\frac{1}{2}}. \end{aligned}\right\} \quad (38)$$

3.2 *The experimental results for unsteady states*

We make the experiments for an unsteady state condition following the experiments for the steady state condition. As the velocity u varies as

$$u = U_0 + U_1 \cos \omega t = U_0(1 + \varepsilon \cos \omega t),$$

we use the Re number based on the time-averaged velocity $\bar{u}$ as the reference velocity. The relation between $\overline{Re}$ and $Re_0 = U_0 D/\nu$ is expressed as follows:

$$\overline{Re} = Re_0\left[\frac{1}{\pi}\int_0^{\pi} |1 + \varepsilon \cos \theta|\, d\theta\right] \equiv Re_0 b_0^2 \quad (39)$$

where the absolute value inside the integral sign means that heat transfer is independent

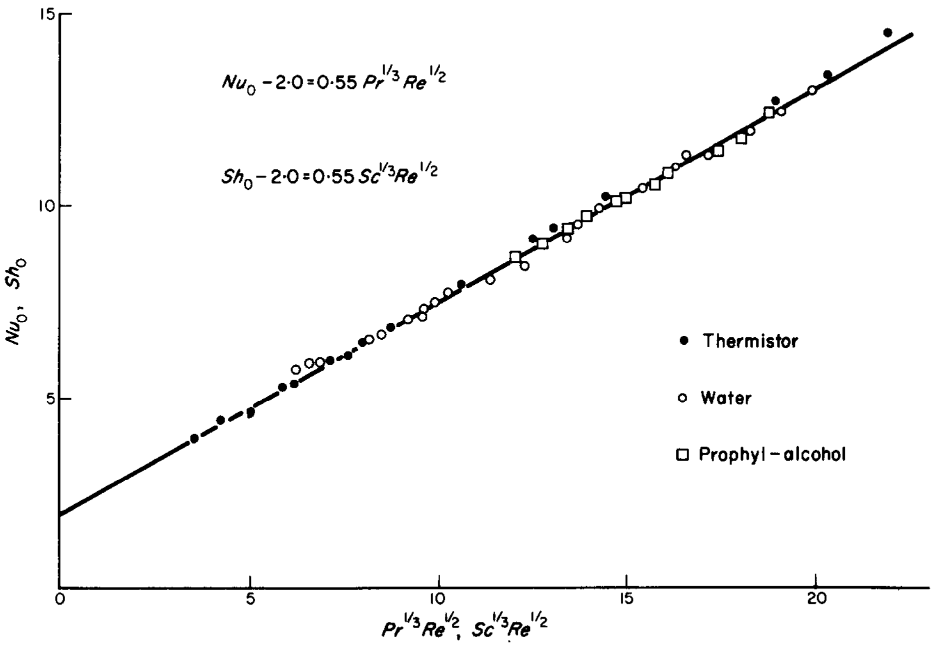

FIG. 7. Steady experimental results for heat and mass transfer from spheres.

of the velocity direction. The value of b_0 is given by

$$\left.\begin{aligned} &\varepsilon \leqslant 1, \quad b_0^2 = 1 \\ &\varepsilon > 1, \quad b_0^2 = \frac{2}{\pi}\left[\sin^{-1}\frac{1}{\varepsilon} + \sqrt{(\varepsilon^2 - 1)}\right]. \end{aligned}\right\} \quad (40)$$

In the region where the S number is very small, at every moment the steady state is considered to be instantly reached, i.e. it may be assumed to be a quasi-steady state. Then, the instantaneous Nu number Nu_{vi} is expressed by use of the equation for the steady state as follows:

$$Nu_{vi} - 2 = mRe_0^{\frac{1}{2}}(|1 + \varepsilon \cos \omega t|)^{\frac{1}{2}}. \quad (41)$$

Then, the time-averaged Nu number is given by

$$Nu_v - 2 = mRe_0^{\frac{1}{2}}\left[\frac{1}{\pi}\int_0^{\pi} (|1 + \varepsilon \cos \theta|)^{\frac{1}{2}}\, d\theta\right] \equiv mRe_0^{\frac{1}{2}} a_0 \quad (42)$$

where the value a_0 is calculated from the following equation.

τ_0 is the angle at the point where the velocity changes its sign and is given by

$$\tau_0 = \frac{\pi}{2} + \sin^{-1}\frac{1}{\varepsilon}.$$

Replacing Re_0 with $\overline{Re}$ by using equation (39), equation (42) is expressed as follows:

$$Nu_v - 2 = m\overline{Re}^{\frac{1}{2}}\frac{a_0}{b_0}. \quad (45)$$

$\overline{Nu}$ is obtained from the following equation by using equation (38) for the steady state:

$$\overline{Nu} - 2 = m\overline{Re}^{\frac{1}{2}}. \quad (46)$$

From equations (45) and (46):

$$\frac{Nu_v - 2}{\overline{Nu} - 2} = \frac{a_0}{b_0} \quad (47)$$

where a_0 and b_0 are given by equations (43) and (40), respectively. In the case of mass transfer, similar analysis can be adopted. By

$$\left.\begin{aligned} &\varepsilon < 1, \quad a_0 = 1 + \sum_{K=1} \frac{(-1)^{2K-1}}{(2K)!\,2^{2K}}(4K-3)!!\,\varepsilon^{2K}\frac{(2K-1)!!}{(2K)!!} \\ &\varepsilon = 1, \quad a_0 = 2\sqrt{(2)}/\pi \\ &\varepsilon > 1, \quad a_0 = \frac{2\sqrt{(\varepsilon+1)}}{\pi}\frac{\tau_0}{2} + \frac{2\sqrt{(\varepsilon-1)}}{\pi}\left[\frac{\pi}{2} - \frac{\tau_0}{2}\right] \\ &\qquad - \frac{2\sqrt{(\varepsilon+1)}}{\pi}\sum_{K=1}\frac{(2K-3)!!}{K!}\left(\frac{\varepsilon}{\varepsilon+1}\right)^K\frac{(-1)^K}{2^{2K}} I \\ &\qquad - \frac{2\sqrt{(\varepsilon-1)}}{\pi}\sum_{K=1}\frac{(2K-3)!!}{K!}\left(\frac{\varepsilon}{\varepsilon-1}\right)^K\frac{1}{2^{2K}} J \end{aligned}\right\} \quad (43)$$

where

$$\left.\begin{aligned} I &= \sum_{r=0}^{K-1}(-1)^r\binom{2K}{r}\frac{\sin \tau_0(K-r)}{(K-r)} + (-1)^K\binom{2K}{K}\frac{\tau_0}{2} \\ J &= -\sum_{r=0}^{K-1}\binom{2K}{r}\frac{\sin \tau_0(K-r)}{(K-r)} + \binom{2K}{K}\left(\frac{\pi}{2} - \frac{\tau_0}{2}\right). \end{aligned}\right\} \quad (44)$$

replacing Nu_v and $\overline{Nu}$ of equation (47) with Sh_v and $\overline{Sh}$ respectively, we obtain

$$\frac{Sh_v - 2}{\overline{Sh} - 2} = \frac{a_0}{b_0}.$$

In Fig. 8 the experimental results of heat transfer from sphere are shown, where the horizontal axis is ε and the parameter is SRe_0. No variation

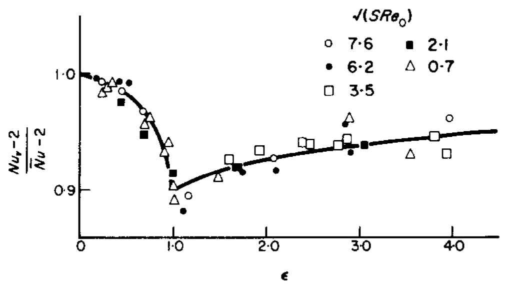

FIG. 8. Unsteady heat transfer with uniform velocity.

of experimental values with SRe_0 can be seen. The solid line shows the theoretical value expressed by equation (47) which is calculated on the assumption of the quasi-steady state. When ε increases, a_0/b_0 in equation (47) tends closer to a constant value. At the limiting case when $\varepsilon \to \infty$, we have no steady component and Nu_v is given by the following equation from equation (47):

$$Nu_v = 2 + 0{\cdot}478\overline{Re}^{\frac{1}{2}}. \qquad (48)$$

The results are shown in Fig. 9 where the open circles show the measured results for the perfect sinusoidal waves, on the other hand, the solid circles are the results for the waves disturbed near the maximum amplitude, when the increase of the velocity amplitude brings the high frequency waves. Thus in the perfect sinusoidal wave, the theoretical values and the experimental ones coincide very well, so the assumption of the quasi-steady state is valid in this experiment. When we have only the fluctuant velocity component, it is theoretically proved that from the stand point of the analysis of flow and heat transfer the case of the fluctuant flow

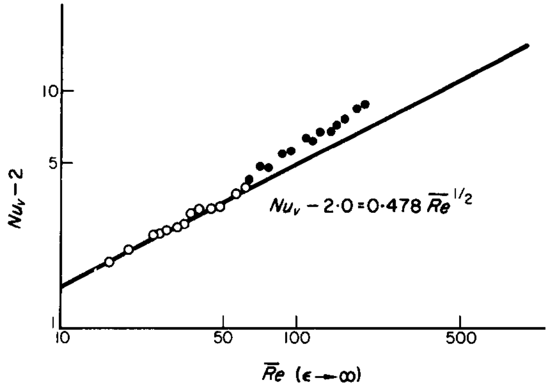

FIG. 9. Unsteady heat transfer without uniform velocity.

field is the same as that when the flow is stopped and the sample is in oscillation. The results of our analysis coincide well with the measured results using the fine vibrating wire by Mabuchi [7], when the effect of natural convection is negligible. The detailed study about it will be reported in our next paper.

The results of mass transfer from a liquid drop are shown in Fig. 10. The solid line is calculated value using the assumption of a quasi-steady state. The effect of the Sc number is not seen. As the oscillation of the liquid drop becomes intense for $\varepsilon > 1$, the accurate experimental results are not obtained.

An accurate analysis of the effect of the velocity

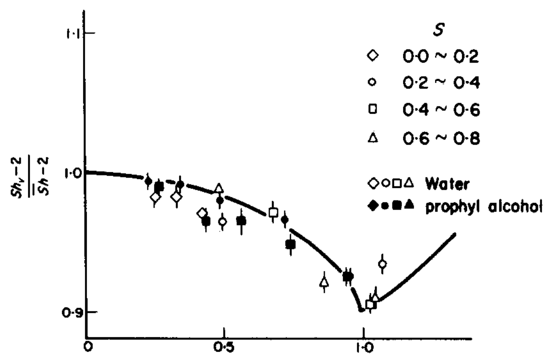

FIG. 10. Unsteady mass transfer with uniform velocity.

fluctuation on the time-averaged Nu number has never been done. However, the local Nu number Nu_{vX} in an unsteady state along the surface of a body is written in terms of Nu number in a steady state, as follows for $\varepsilon < 1, S < 1$:

$$Nu_{vX} = Nu_{sX}[1 + O(\varepsilon^2) + O(\varepsilon^2 S_X^2) + O(\varepsilon^4) + \ldots.] \quad (49)$$

where X is the distance from the stagnation point along the surface, $S_X = \omega D/V(X)$ is the local Strouhal number, ε is non-dimensional amplitude of the fluctuant velocity and $V(X)$ the velocity outside the boundary layer at X. $V(X)$ is expressed as follows:

$$\frac{V}{U_0} = C_1\left(\frac{X}{D}\right) + C_3\left(\frac{X}{D}\right)^3 \ldots. \quad (50)$$

Within the Re number region of this experiment, V is approximated by the first term of equation (50), so S_X is written as follows:

$$S_X = \frac{\omega D}{V} \doteqdot \frac{1}{C_1}\frac{\omega D}{U_0} = \frac{S}{C_1} \quad (51)$$

where C_1 is equal to 3·0 for the sphere. In this experiment, S_X is smaller than 0·2, therefore, the terms containing S_X of equation (49) are negligible. This is the reason why the experimental values are independent of Strouhal number and almost agree with the theoretical value calculated under the assumption of the quasi-steady state.

4. THE VIBRATION OF THE LIQUID DROP

The experimental results of mass transfer from liquid drops are obtained under the condition that the liquid drop is spherical. At a certain frequency of the fluctuant velocity, a liquid drop makes a transforming oscillation as shown in Fig. 11. The liquid drop has a natural frequency of transforming vibration caused by surface tension in a gas or in a different liquid. It seems that the liquid drop shown in Fig. 11 makes a resonance of the natural frequency with that of the fluctuant velocity.

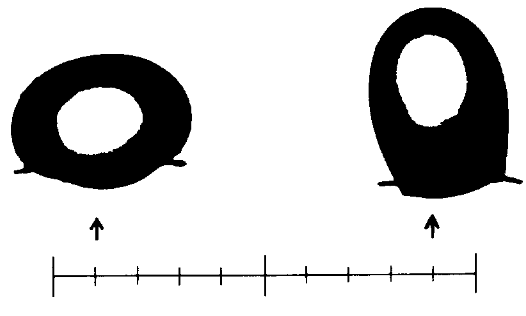

FIG. 11. Droplet transformation in resonant state.

4.1 *The natural frequency of the liquid drop*

We take an origin of a co-ordinates at the center of the gravity of the drop shown in Fig. 12. We assume that the velocity potential ϕ varies in the following sinusoidal way of angular velocity ω.

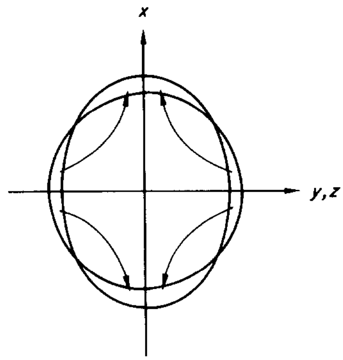

FIG. 12. Model of transforming vibration of droplet.

$$\phi = [x^2 - \tfrac{1}{2}(y^2 + z^2)]\, a \cos \omega t. \quad (52)$$

When x_0, y_0, z_0 are the coordinates of the surface in the case of no vibration, the velocity and the displacement in x, y, z direction are expressed as:

$$u = \frac{\partial \phi}{\partial x} = 2ax \cos \omega t = \frac{\mathrm{d}x}{\mathrm{d}t} \quad (53)$$

$$x = x_0 \exp\left\{\frac{2a}{\omega} \sin \omega t\right\} \quad (54)$$

$$v = \frac{\partial \phi}{\partial y} = -ay\cos\omega t = \frac{dy}{dt} \quad (55)$$

$$y = y_0 \exp\left\{-\frac{a}{\omega}\sin\omega t\right\} \quad (56)$$

$$w = \frac{\partial \phi}{\partial z} = -az\cos\omega t = \frac{dz}{dt} \quad (57)$$

$$z = z_0 \exp\left\{-\frac{a}{\omega}\sin\omega t\right\}. \quad (58)$$

By using the relation; $x_0^2 + y_0^2 + z_0^2 = R_0^2$ and replacing $(a/\omega)\sin\omega t = g(\omega)$, from equations (54), (56) and (58), the following equation is given:

$$\frac{x^2}{e^{4g}} + \frac{y^2}{e^{-2g}} + \frac{z^2}{e^{-2g}} = R_0^2. \quad (59)$$

In order to obtain the potential energy due to the surface tension, we calculate the surface area F of the liquid drop from equation (59). By making the transformation of the variables below,

$$x = R_0 e^{2g}\sin\varphi, \qquad \eta = R_0 e^{-g}\cos\varphi \quad (60)$$

the surface area is given by:

$$F = 2\pi \int_{-\pi/2}^{\pi/2} R_0^2 e^{g} \sqrt{(\cos^2\varphi + e^{-6g}\sin^2\varphi)} \times \cos\varphi \, d\varphi$$

$$= 2\pi R_0^2 e^{g}\left[e^{-3g} - \frac{\sin^{-1}\sqrt{(1-e^{-6g})}}{\sqrt{(1-e^{-6g})}}\right]. \quad (61)$$

The deformation of the liquid drop from the sphere being small, the potential energy V due to the surface tension σ is given as follows:

$$V = \sigma F \doteqdot 4\pi R_0^2\sigma[1 + \tfrac{8}{5}g^2]. \quad (62)$$

The kinetic energy of the liquid drop T_1 is represented as:

$$T_1 = \frac{\rho}{2}\int_{(V)} (u^2 + v^2 + w^2)\, dx dy dz \quad (63)$$

$$= \frac{2\pi\rho a^2 R_0^5 \cos^2\omega t}{15}[2e^{-2g} + 4e^{4g}] \quad (64)$$

where ρ is the density of the liquid. Expanding equation (64) in power series and taking the largest term, T_1 is given by

$$T_1 \doteqdot \frac{4\pi R_0^5 \rho a^2 \cos^2\omega t}{5}. \quad (65)$$

As $T_1 + V$ is constant, the time derivative $d(T_1 + V)/dt$ vanishes. Then the natural angular velocity of natural transforming vibration of the liquid drop is given by the following equation:

$$\omega_{\mathrm{I}}^2 = \frac{8\sigma}{\rho R_0^3}. \quad (66)$$

If one end of the liquid drop is fixed, the velocity of the x direction is equal to equation (53) added to the velocity of the center of the gravity and is given as:

$$u' = 2(r\sin\varphi + R_0)e^{2g}a\cos\omega t.$$

By making the same calculation as before, the kinetic energy in this case is expressed as follows:

$$T_{\mathrm{II}} = \frac{52\pi R_0^5 \rho a^2 \cos^2\omega t}{15}. \quad (67)$$

Then, the angular velocity of the natural vibration is given by

$$\omega_{\mathrm{II}}^2 = \frac{1{\cdot}85}{R_0^3}\frac{\sigma}{\rho} = \frac{14{\cdot}8}{D^3}\frac{\sigma}{\rho}. \quad (68)$$

In the experiment of mass transfer for an unsteady flow, the diameter of the drop decreases with time even in a resonance state. Therefore, we get the relation between the mean diameter before and after the resonance state and the frequency of the vibration of the drop, and show it in Fig. 13. The solid line is the calculated value by equation (68). The difference of the experimental values and the theoretical values is about ten per cent. If we think the fact that in the theory the effect of the supporting wires is neglected, that is, the liquid drop is assumed to be supported at one point, the agreement between the theory and experiment is considered to be satisfactory.

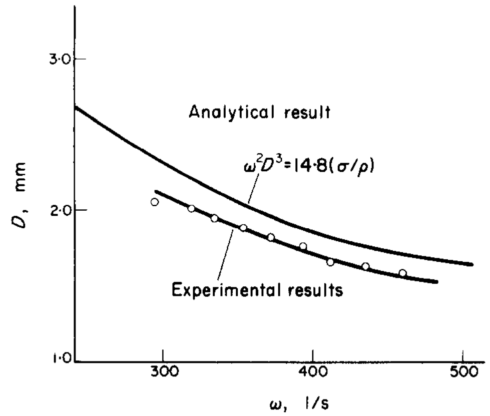

FIG. 13. Frequency of resonant vibration of droplet.

4.2 *The mass-transfer coefficient at the resonance state*

The increase of the average mass-transfer coefficient is obtained when the liquid drop makes the resonant transforming vibration stated above. These results are shown in Fig. 14. The horizontal axis is $\varepsilon Re_0 = U_1 D/\nu$ (U_1 is the amplitude of the fluctuant velocity), so called the *Re* number of the vibration, and the vertical axis is represented by a_0/b_0. The results show that a_0/b_0 is independent of the *Re* number Re_0 and increases proportionally to εRe_0, i.e. the *Re* number of the vibration in

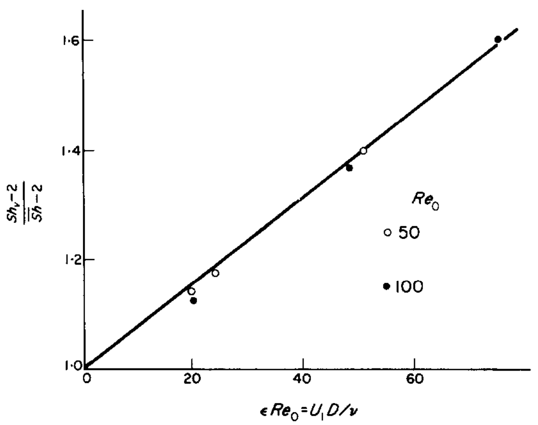

FIG. 14. Unsteady experimental results for mass transfer from resonating droplets.

contrast with the non-resonance state. This is due to the fact that the flow around the liquid drop is changed and the surface area of the drop increases by the vibration of the drop.

5. CONCLUSION

This paper makes the experimental and theoretical studies on heat and mass transfer coefficients from small spheres for an unsteady flow, in which the velocity u is vibrated as $u = V_0(1 + \varepsilon \cos \omega t)$ and the following conclusions are obtained:

(1) The experimental results of the time-average *Nu* number of heat transfer for the unsteady flow coincide well with the calculated values under the assumption of the quasi-steady state. In this experimental region of the Strouhal number *S*, there is no variation of the *Nu* number by the *S* number, and the *Nu* number only depends on the *Re* number taking the time-average velocity as the reference and the dimensionless velocity amplitude ε. This relation can also be adopted in the case where there is no uniform flow, only the fluctuant velocity. The time-averaged *Nu* number Nu_v is generally smaller than the *Nu* number $\overline{Nu}$ of steady heat transfer for the time-averaged velocity $\bar{u}$. When the amplitude of the fluctuant velocity is equal to the uniform velocity, $(Nu_v - 2)/(\overline{Nu} - 2)$, $(= a_0/b_0)$ is the minimum value 0·900, and when there is no uniform flow and only the fluctant velocity, it is 0·958.

(2) The Sherwood number for mass transfer from the liquid drop of water or prophyl-alcohol has the same relationship as that of the *Nu* number, it only depends on Re_0 and ε.

(3) It is theoretically and experimentally verified that the transforming vibration of the liquid drop yielded during the experiment under the unsteady state condition is caused by the resonance; i.e. the coincidence of the natural frequency of the liquid drop due to the surface tension with the frequency of the fluctuant velocity. Also, in such a case, the mass-transfer coefficient is discovered to increase with the *Re* number of the vibration.

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Résumé—Cet article expose les résultats théoriques et expérimentaux sur le transport de chaleur et de masse instationnaire à partir de petits corps sphériques pour de petits nombres de Strouhal. Des expériences sont faites dans un écoulement ayant des composantes permanente et instationnaire. Les écoulements instationnaires sont engendrés par une résonance acoustique dans un tube et fluctuent d'une façon purement sinusoïdale. Au cours d'expériences de transport de chaleur dans un écoulement d'air, plusieurs thermistances sphériques sont utilisées et l'on montre que la moyenne temporelle du nombre de Nusselt est bien corrélée, à la fois expérimentalement et théoriquement, au moyen du nombre de Reynolds basé sur la moyenne temporelle de la vitesse et du rapport de l'amplitude de vitesse de la composante instationnaire à la composante stationnaire.

Les expériences de transport de masse instationnaire sont effectuées par évaporation de petites goutelettes d'eau et d'alcool propylique dans un écoulement d'air. On trouve que les résultats sont représentés par une corrélation semblable à celle du transport de chaleur. On observe une oscillation résonante pour la déformation d'une gouttelette due à la tension superficielle et l'on trouve que pendant cette résonance le nombre de Sherwood croît proportionnellement à la composante instationnaire de l'écoulement.

Zusammenfassung—Diese Arbeit berichtet über theoretisch und experimentell gewonnene Ergebnisse bei instationärem Wärme- und Stoffübergang von kleinen kugeligen Körpern bei kleinen Strouhal-Zahlen. Die Experimente wurden in einer Strömung mit stationären und instationären Komponenten durchgeführt. Die rein sinusförmig schwingende instationäre Strömung wurde durch akustische Resonanz in einem Rohr hergestellt. Bei den Wärmeübergangsexperimenten in einem Luftstrom wurden kugelige Thermistoren verwendet; für die zeitlich gemittelte Nusseltzahl ergibt sich sowohl theoretisch als auch experimentell eine klare Abhängigkeit von der Reynolds-Zahl, gebildet mit der zeitlich gemittelten Geschwindigkeit, und von dem Verhältnis der Amplitude der instationären Geschwindigkeit zu deren stationärem Anteil.

Die instationären Stofftransportversuche wurden durch Verdampfen kleiner Wasser- und Prophyl-Alkoholtröpfchen in Luft durchgeführt. Es zeigte sich, dass sich diese Ergebnisse in einer dem Wärmeübergang ähnlichen Beziehung darstellen lassen. Dabei wurde eine Deformation der Tröpfchen nach einer Resonanzschwingung beobachtet, die durch die Oberflächenspannung verursacht war. Während dieser Resonanz wächst die Sherwood-Zahl proportional dem instationären Anteil der Strömung.

Аннотация—В статье приводятся теоретические и экспериментальные результаты по нестационарному тепло-и массопереносу от небольших сферических тел при малых значениях критерия Струхаля. Опыты проводились в потоке, состоящем из стационарного и нестационарного компонентов. Нестационарные потоки генерировались с помощью акустического резонанса в трубе и были чисто синусоидальными. В опытах по теплообмену в потоке воздуха использовались несколько сферических термистров. Показано, что осредненный по времени критерий Нуссельта хорошо коррелируется экспериментально и теоретически с помощью критерия Рейнольдса, определенного по осредненной по времени скорости, и отношения амплитуды скорости нестационарного компонента к стационарному.

Эксперименты по нестационарному переносу массы были выполнены с помощью испарения малых капель воды и пропилового спирта в потоке воздуха. Наблюдается резонирующее колебание деформации капельки. Найдено, что во время этого резонанса критерий Шервуда возрастает пропорционально нестационарному компоненту потока.